

Class IX Session 2025-26
Subject - Mathematics
Sample Question Paper - 5

Time Allowed: 3 hours

Maximum Marks: 80

General Instructions:

Read the following instructions carefully and follow them:

1. This question paper contains 38 questions.
2. This Question Paper is divided into 5 Sections A, B, C, D and E.
3. In Section A, Questions no. 1-18 are multiple choice questions (MCQs) and questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21-25 are very short answer (VSA) type questions, carrying 02 marks each.
5. In Section C, Questions no. 26-31 are short answer (SA) type questions, carrying 03 marks each.
6. In Section D, Questions no. 32-35 are long answer (LA) type questions, carrying 05 marks each.
7. In Section E, Questions no. 36-38 are case study-based questions carrying 4 marks each with sub-parts of the values of 1,1 and 2 marks each respectively.
8. All Questions are compulsory. However, an internal choice in 2 Questions of Section B, 2 Questions of Section C and 2 Questions of Section D has been provided. An internal choice has been provided in all the 2 marks questions of Section E.
9. Draw neat and clean figures wherever required.
10. Take $\pi = 22/7$ wherever required if not stated.
11. Use of calculators is not allowed.

Section A

1. The name of the vertical line drawn to determine the position of any point in the Cartesian plane is **[1]**
 - a) x-axis
 - b) Cartesian line
 - c) y-axis
 - d) Origin
2. The perimeter of a field in the form of an equilateral triangle is 36 cm, then its area is given by **[1]**
 - a) $36\sqrt{3} \text{ cm}^2$
 - b) $8\sqrt{3} \text{ cm}^2$
 - c) $98\sqrt{3} \text{ cm}^2$
 - d) $42\sqrt{3} \text{ cm}^2$
3. In the given figure, BOC is a diameter of a circle and $AB = AC$. Then, $\angle ABC = ?$ **[1]**



-
- A right-angled triangle ABC is shown with the right angle at vertex B . Point D is on side AB and point E is on side AC . A line segment DE is drawn parallel to BC . The medians BE and CD intersect at point O .

6. In figure, for which value of x is $l_1 \parallel l_2$? [1]



8. Which of the following expressions is a polynomial? [1]

9. Value of $(256)^{0.16} \times (256)^{0.09}$ is **[1]**

- a) 256.25 b) 16
c) 64 d) 4

10. ABCD is a Parallelogram in which AB = 9.5 cm and its perimeter is 30 cm. Find the length of each side of the Parallelogram? [1]

- a) 9.5 cm, 9.5 cm, 5.4 cm, 5.6 cm b) 9.5 cm, 9.5 cm, 5.6 cm, 5.4 cm
c) 10 cm, 10 cm, 11 cm, 11 cm d) 9.5 cm, 9.5 cm, 5.5 cm, 5.5 cm

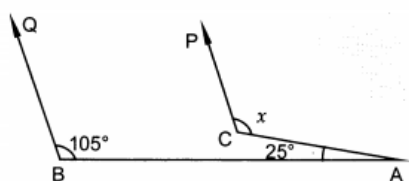
11. The value of $\frac{2}{\sqrt{5}-\sqrt{3}}$ is [1]

- a) $\sqrt{5} + \sqrt{3}$ b) $\frac{1}{\sqrt{5}-\sqrt{3}}$
c) $\sqrt{5} - \sqrt{3}$ d) $\frac{1}{\sqrt{5}+\sqrt{3}}$

12. The graph of the linear equation $y = x$ passes through the point [1]

- a) $\left(\frac{3}{2}, \frac{-3}{2}\right)$ b) $\left(\frac{-1}{2}, \frac{1}{2}\right)$
c) (1, 1) d) $\left(0, \frac{3}{2}\right)$

13. In Fig. if $CP \parallel BQ$, then the measure of x is [1]

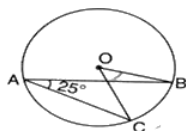


- a) 130° b) 105°
c) 175° d) 125°

14. How many digits are there in the repeating block of digits in the decimal expansion of $\frac{17}{7}$? [1]

- a) 16 b) 26
c) 7 d) 6

15. In the given figure, if $\angle BAC = 25^\circ$, then $\angle BOC$ is equal to [1]



- a) 25° b) 125°
c) 60° d) 50°

16. The points A(-2, 3), B(-2, -4) and C(5, -4) are the vertices of the square ABCD, then the co-ordinates of the vertex D are: [1]

- a) (5, 3) b) (3, 3)
c) (0, 0) d) (3, -4)

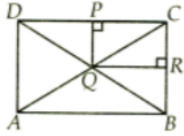
17. The point which lies on x-axis at a distance of 3 units in the positive direction of x-axis is [1]

- a) (3, 0) b) (0, -3)
c) (-3, 0) d) (0, 3)

18. If $a^2 + b^2 + c^2 - ab - bc - ca = 0$, then [1]

- a) $c + a = b$ b) $a + b = c$
c) $b + c = a$ d) $a = b = c$

19. **Assertion (A):** ABCD and PQRC are rectangles and Q is a midpoint of AC. Then DP = PC. [1]



Reason (R): The line segment joining the midpoint of any two sides of a triangle is parallel to the third side and equal to half of it.

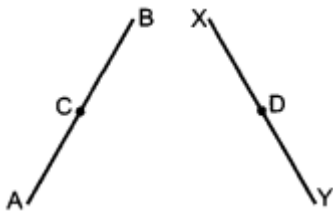
- a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A.
- c) A is true but R is false. d) A is false but R is true.
20. **Assertion (A):** e is an irrational number. [1]

Reason (R): π is an irrational number.

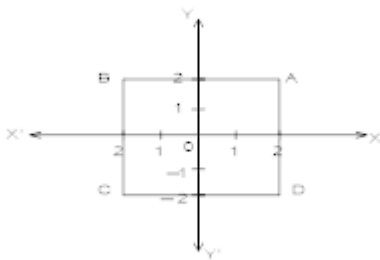
- a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A.
- c) A is true but R is false. d) A is false but R is true.

Section B

21. In fig. AC = XD, C is the mid-point of AB and D is the mid-point of XY. Using a Euclid's axiom, show that AB = XY. [2]



22. If P, Q, and R are three points on a line and Q is between P and R, then prove that $PR - QR = PQ$. [2]
23. Find Co-ordinates of vertices of rectangle ABCD. [2]



24. Give two rational numbers between 0.51511511151115... and 0.5353353335 ... [2]

OR

Simplify: $\frac{\frac{1}{9} \times 27^{\frac{1}{2}}}{\frac{1}{3} \times 3^{\frac{2}{3}}}$.

25. The radius of a spherical balloon increases from 7 cm to 14 cm as air is being pumped into it. Find the ratio of surface areas of the balloon in the two cases. [2]

OR

If the heights of two right circular cones are in the ratio 1 : 2 and the perimeters of their bases are in the ratio 3 : 4, what is the ratio of their volumes?

Section C

26. Represent $\sqrt{9.3}$ on the number line. [3]
27. Given below are the seats won by different political parties in the polling outcome of a state assembly elections: [3]



Political party	A	B	C	D	E	F
Seats won	75	55	37	29	10	37

- Draw a bar graph to represent the polling results.
 - Which political party won the maximum number of seats?
28. E is the mid-point of the side AD of the trapezium ABCD with $AB \parallel DC$. A line through E is drawn parallel to AB intersect BC at F. Show that F is the mid-point of BC. [hint: Join AC] [3]
29. Draw the graph of the following linear equation in two variables: $x + y = 4$ [3]
30. Draw a histogram of the following distribution: [3]

Height (in cm)	Number of students
150 - 153	7
153 - 156	8
156 - 159	14
159 - 162	10
162 - 165	6
165 - 168	5

OR

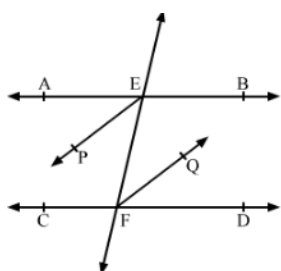
100 surnames were randomly picked up from a local telephone directory and a frequency distribution of the number of letters in the English alphabets in the surnames was found as follows :

Number of letters	Number of surnames
1-4	6
4-6	30
6-8	44
8-12	16
12-20	4

- Draw a histogram to depict the given information.
 - Write the class interval in which the maximum number of surnames lie.
31. The polynomials $ax^3 + 3x^2 - 3$ and $2x^3 - 5x + a$ when divided by $(x - 4)$ leave the remainders R_1 and R_2 respectively. Find the values of a if $R_1 = R_2$ [3]

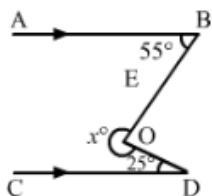
Section D

32. In the given figure, $AB \parallel CD$ and a transversal t cuts them at E and F respectively. If EP and FQ are the bisectors of $\angle AEF$ and $\angle EFD$ respectively, prove that $EP \parallel FQ$. [5]

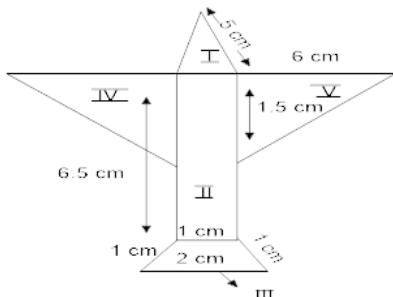


OR

In each of the figures given below, $AB \parallel CD$. Find the value of x°



33. A well of inner diameter 14 m is dug to a depth of 15 m. Earth taken out of it has been evenly spread all around it to a width of 7 m to form an embankment. Find the height of the embankment so formed. [5]
34. Radha made a picture of an aeroplane with colored paper as shown in fig. find the total area of the paper used. [5]



OR

The perimeter of a right triangle is 24 cm. If its hypotenuse is 10 cm, find the other two sides. Find its area by using the formula area of a right triangle. Verify your result by using Heron's formula.

35. Find the values of m and n so that the polynomial $x^3 - mx^2 - 13x + n$ has $x-1$ and $x+3$ as factors. [5]

Section E

36. Read the following text carefully and answer the questions that follow: [4]

Ajay is writing a test which consists of 'True' or 'False' questions. One mark is awarded for every correct answer while $\frac{1}{4}$ mark is deducted for every wrong answer. Ajay knew answers to some of the questions. Rest of the questions he attempted by guessing.



- If he answered 110 questions and got 80 marks and answer to all questions, he attempted by guessing were wrong, then how many questions did he answer correctly? (1)
- If he answered 110 questions and got 80 marks and answer to all questions, he attempted by guessing were wrong, then how many questions did he guess? (1)
- If answer to all questions he attempted by guessing were wrong and answered 80 correctly, then how many marks he got? (2)

OR

If answer to all questions he attempted by guessing were wrong, then how many questions answered correctly to score 95 marks? (2)

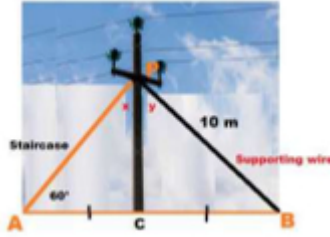
37. Read the following text carefully and answer the questions that follow: [4]

As shown In the village of Surya there was a big pole PC. This pole was tied with a strong wire of 10 m length. Once there was a big spark on this pole, thus wires got damaged very badly. Any small fault was usually



repaired with the help of a rope which normal board electricians were carrying on bicycles.

This time electricians need a staircase of 10 m so that it can reach at point P on the pole and this should make 60° with line AC.



- Show that $\triangle APC$ and $\triangle BPC$ are congruent. (1)
- Find the value of $\angle x$. (1)
- Find the value of $\angle y$. (2)

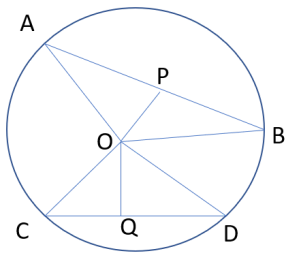
OR

What is the value of $\angle PBC$? (2)

38. **Read the following text carefully and answer the questions that follow:**

[4]

Rohan draws a circle of radius 10 cm with the help of a compass and scale. He also draws two chords, AB and CD in such a way that the perpendicular distance from the center to AB and CD are 6 cm and 8 cm respectively. Now, he has some doubts that are given below.



- Show that the perpendicular drawn from the Centre of a circle to a chord bisects the chord. (1)
- What is the length of CD? (1)
- What is the length of AB? (2)

OR

How many circles can be drawn from given three noncollinear points? (2)

Solution

Section A

1.
(c) y-axis
Explanation:
In the Cartesian plane, there are two axes. One is a horizontal line, that is called the x-axis and other is a vertical line, that is called y-axis.
2. (a) $36\sqrt{3} \text{ cm}^2$
Explanation:
Since all the sides are equal in an equilateral triangle.
So, perimeter = $a + a + a$, where a is the side of equilateral triangle.
 $\Rightarrow 3a = 36 \Rightarrow a = 12 \text{ cm}$
 \therefore Area of equilateral triangle = $\frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4}(12)^2 = \frac{\sqrt{3}}{4} \times 144$
 $= 36\sqrt{3} \text{ cm}^2$
3.
(d) 45°
Explanation:
Since an angle in a semicircle is a right angle, $\angle BAC = 90^\circ$
 $\therefore \angle ABC + \angle ACB = 90^\circ \dots(1)$
Now, $AB = AC$ (Given)
 $\Rightarrow \angle ABC = \angle ACB = 45^\circ \dots(2)$
 $\Rightarrow \angle ABC + \angle ACB = 90^\circ$ [From (1) and (2)]
 $\Rightarrow 2\angle ABC = 90$
 $\Rightarrow \angle ABC = 45^\circ$
4. (a) 13.5 cm^2
Explanation:
As D and E are the midpoints of AB and AC.
So, by mid point theorem
 $DE = BC/2 = 12/2 = 6 \text{ cm}$
 $AD = AB/2 = 9/2 = 4.5 \text{ cm}$
Area of $\triangle ADE = 0.5 \times DE \times AD$
 $= 0.5 \times 6 \times 4.5 = 13.5 \text{ cm square}$
5. (a) 486
Explanation:
 $9^3 + (-3)^3 - 6^3$
 $= 729 - 27 - 216$
 $= 729 - 243$
 $= 486$
6.
(c) 47
Explanation:
Let if $l_1 \parallel l_2$ and AB is tranverse to it
Then,
 $\angle PBA$ should be equal to $\angle BAS$ (Alternate angles)



So if $l_1 \parallel l_2$, then $\angle BAS = 70^\circ$

$$\Rightarrow \angle BAC = 78^\circ - 35^\circ = 43^\circ \text{..(i)}$$

Now, in $\triangle ABC$

$$x^\circ + \angle C + \angle BAC = 180^\circ$$

$$\Rightarrow x^\circ + 90^\circ + 43^\circ = 180^\circ$$

$$\Rightarrow x^\circ = 180^\circ - 90^\circ - 43^\circ = 47^\circ$$

$$\Rightarrow x^\circ = 47^\circ$$

So if $x^\circ = 47^\circ$ then $l_1 \parallel l_2$

7. (a) 4

Explanation:

Given, (4, 19) is a solution of the equation $y = ax + 3$

$$= 19 = 4a + 3$$

$$= a = 4$$

8. (a) $x^2 + \frac{2x^{3/2}}{\sqrt{x}} + 6$

Explanation:

$$\text{We have: } x^2 + \frac{2x^{3/2}}{\sqrt{x}} + 6 = x^2 + 2x^{3/2} x^{-1/2} + 6$$

$$= x^2 + 2x + 6$$

It is a polynomial because it has only non-negative integral powers of x

9.

(d) 4

Explanation:

$$(256)^{0.16} \times (256)^{0.09}$$

$$= (256)^{0.16 + 0.09}$$

$$= (256)^{0.25}$$

$$= [(2)^8]^{\frac{1}{4}}$$

$$= (2)^2$$

$$= 4$$

10.

(d) 9.5 cm, 9.5 cm, 5.5 cm, 5.5 cm

Explanation:

$$\text{Perimeter of ABCD} = AB + BC + CD + DA = 30$$

In a parallelogram, opposite sides are equal.

$$AB = CD = 9.5 \text{ and } BC = DA = x$$

$$\text{So, } 9.5 + x + 9.5 + x = 30$$

$$2x = 30 - 19$$

$$x = 5.5$$

$$AB = 9.5 = CD \text{ and } BC = DA = 5.5$$

11. (a) $\sqrt{5} + \sqrt{3}$

Explanation:

$$\frac{2}{\sqrt{5} - \sqrt{3}}$$

multiplying numerator and denominator by

$$\sqrt{5} + \sqrt{3}, \text{ we get}$$

$$\frac{2(\sqrt{5} + \sqrt{3})}{(\sqrt{5} - \sqrt{3})(\sqrt{5} + \sqrt{3})}$$

$$= \frac{2(\sqrt{5} + \sqrt{3})}{5 - 3} = \sqrt{5} + \sqrt{3}$$

12.

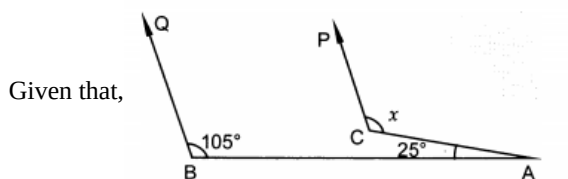
(c) (1, 1)

Explanation:

$y = x$, \Rightarrow both the coordinates are the same. Hence (1, 1) is correct option.

13. (a) 130°

Explanation:



$CP \parallel BQ$

Produce CP to E

So, $PE \parallel BQ$ and AB cuts them

$\angle QBE = \angle QBA = 105^\circ$ (Corresponding angles)

In $\triangle ECA$

$\angle CEA + \angle ECA + \angle EAC = 180^\circ$

$105^\circ + \angle ECA + 25^\circ = 180^\circ$

$\angle ECA = 50^\circ$

$\angle PCA + \angle ECA = 180^\circ$ (Linear pair)

$x + 50^\circ = 180^\circ$

$x = 130^\circ$

14.

(d) 6

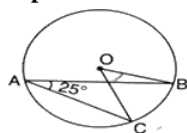
Explanation:

$$\frac{17}{7} = 2.428571$$

15.

(d) 50°

Explanation:



Angle made at centre by an arc is double the angle made by it on any point on the circumference.

16. (a) (5, 3)

Explanation:

Let $A(-2, 3)$, $B(-2, -4)$, $C(5, -4)$ be the three vertices of the square ABCD.

Clearly, the abscissa of D = abscissa of C = 5

And, ordinate of D = ordinate of A = 3

So, the coordinates of the 4th vertex of ABCD i.e. D are (5, 3).

17. (a) (3, 0)

Explanation:

At x-axis the value of y coordinate is 0 and x-axis at a distance of 3 units in the positive direction so the co-ordinate of the x-axis is 3.

So the co-ordinate of point is (3, 0).

18.

(d) $a = b = c$

Explanation:

Given: $a^2 + b^2 + c^2 - ab - bc - ca = 0$

$$\Rightarrow 2(a^2 + b^2 + c^2 - ab - bc - ca) = 0$$

$$\Rightarrow (2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca) = 0$$

$$\Rightarrow (\{a^2 + b^2 - 2ab\} + \{b^2 + c^2 - 2bc\} + \{c^2 + a^2 - 2ca\}) = 0$$

$$\Rightarrow (a - b)^2 + (b - c)^2 + (c - a)^2 = 0$$

Now, since the sum of all squares is zero

$$\Rightarrow a - b = 0 \Rightarrow a = b$$

$$\Rightarrow b - c = 0 \Rightarrow b = c$$

$$\Rightarrow c - a = 0 \Rightarrow c = a$$

$$\Rightarrow a = b = c$$

19.

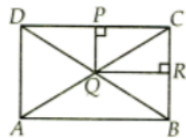
(b) Both A and R are true but R is not the correct explanation of A.

Explanation:

In $\triangle ADC$, Q is the midpoint of AC such that $PQ \parallel AD$.

P is the mid-point of DC

DP = PC [Using converse of midpoint theorem]



20.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation:

Both A and R are true but R is not the correct explanation of A.

Section B

21. In the above figure, we have

$$AB = AC + BC = AC + AC = 2AC \text{ (Since, C is the mid-point of AB) } \dots(1)$$

$$XY = XD + DY = XD + XD = 2XD \text{ (Since, D is the mid-point of XY) } \dots(2)$$

$$\text{Also, } AC = XD \text{ (Given) } \dots(3)$$

From (1),(2)and(3), we get

$AB = XY$, According to Euclid, things which are double of the same things are equal to one another.

22. From the given condition, we get the following figure



In the above figure, PQ coincides with PR - QR.

So, according to Euclid's axiom, "things" which coincide with one another are equal to 'one another'. We have,

$$PQ + QR = PR \text{ i.e. } PR - QR = PQ.$$

23. The co-ordinates of vertices of rectangle A (2, 2), B (-2, 2), C (-2, -2) and D (2, -2). it is a square.

24. Let, $a = 0.51511511151115 \dots = 0.\overline{51}$

$$\text{and, } b = 0.5353353335 \dots = 0.\overline{53}$$

Clearly, $a < b$ since the second decimal place of a has digit 1 and b has digit 3.

Thus two rational numbers lies between $0.\overline{51}$ and $0.\overline{53}$ are 0.52 and 0.522

[Note: Between two irrational numbers there exist infinitely many rational numbers]

OR

$$\begin{aligned}
 \text{Given, } & \frac{9^{\frac{1}{3}} \times 27^{-\frac{1}{2}}}{3^{\frac{1}{6}} \times 3^{-\frac{2}{3}}} \\
 = & \frac{9^{\frac{1}{3}} \times 3^{\frac{2}{3}}}{3^{\frac{1}{6}} \times 27^{\frac{1}{2}}} \\
 = & \frac{(3^2)^{\frac{1}{3}} \times 3^{\frac{2}{3}}}{3^{\frac{1}{6}} \times (3^3)^{\frac{1}{2}}} \\
 = & \frac{3^{\frac{2}{3}} \times 3^{\frac{2}{3}}}{3^{\frac{1}{6}} \times 3^{\frac{3}{2}}} \\
 = & \frac{3^{\frac{2}{3} + \frac{2}{3}}}{3^{\frac{1}{6} + \frac{3}{2}}} \\
 = & \frac{3^{\frac{4}{3}}}{3^{\frac{1+9}{6}}} \\
 = & \frac{3^{\frac{4}{3}}}{3^{\frac{10}{6}}} \\
 = & 3^{\frac{4}{3} - \frac{5}{3}} \\
 = & 3^{-\frac{1}{3}} \\
 = & \frac{1}{3^{\frac{1}{3}}}
 \end{aligned}$$

25. Case I : $r = 7$ cm

$$\begin{aligned}
 \text{Surface area} &= 4\pi r^2 \\
 &= 4 \times \frac{22}{7} \times (7)^2 = 616 \text{ cm}^2
 \end{aligned}$$

Case II : $r = 14$ cm

$$\begin{aligned}
 \text{Surface area} &= 4\pi r^2 \\
 &= 4 \times \frac{22}{7} \times (14)^2 = 2464 \text{ cm}^2
 \end{aligned}$$

\therefore Ratio of surface area of the balloon = 616 : 2464

$$= 1 : 4$$

OR

Ratio in the heights of two cones = 1 : 2 and ratio in the perimeter of their bases = 3 : 4

Let r_1, r_2 be the radii of two cones and h_1 and h_2 be their heights

$$\therefore \frac{h_1}{h_2} = \frac{1}{2}$$

$$\text{and } \frac{2\pi r_1}{2\pi r_2} = \frac{3}{4}$$

$$\Rightarrow \frac{r_1}{r_2} = \frac{3}{4}$$

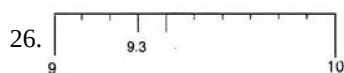
$$\text{Now } \frac{\text{Volume of first cone}}{\text{Volume of second cone}} = \frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2}$$

$$= \frac{r_1^2 h_1}{r_2^2 h_2} = \left(\frac{r_1}{r_2}\right)^2 \times \frac{h_1}{h_2}$$

$$= \left(\frac{3}{4}\right)^2 \times \frac{1}{2} = \frac{9}{16} \times \frac{1}{2} = \frac{9}{32}$$

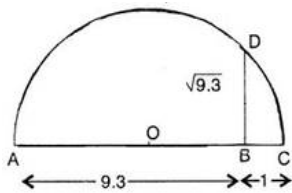
\therefore Ratio in their volumes = 9 : 32

Section C

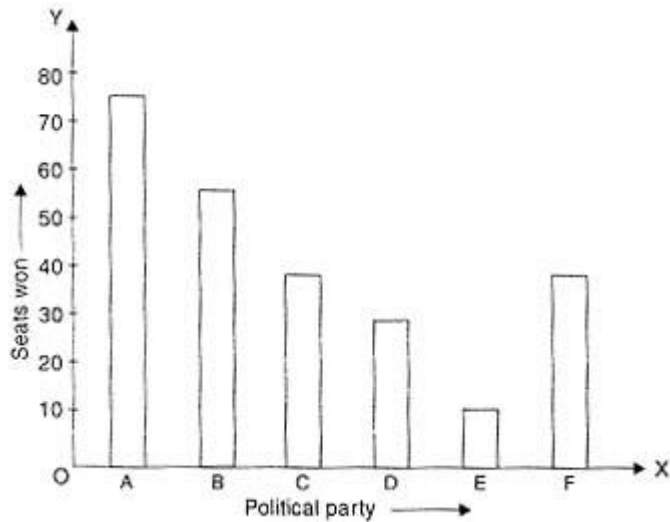


The distance 9.3 from a fixed point A on a given line to obtain a point B such that $AB = 9.3$ units. From B mark a distance of 1 unit and mark the new point as C. Find the mid-point of AC and mark that point as O. Draw a semi-circle with centre O and radius

OC. Draw a line perpendicular to AC passing through B and intersecting the semi-circle at D. Then $BD = \sqrt{9.3}$.

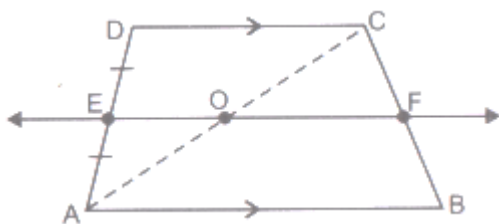


27. i.



ii. Political party A won the maximum number of seats.

28. It is given that in a trapezium ABCD $AB \parallel CD$ and E is the mid-point of the side AD. Also, $EF \parallel AB$. we have to prove that F is the mid-point of BC.



Construction: Join AC which intersects EF at O.

Proof: Here, E is the mid-point of AD and $EF \parallel DC$ [since $EF \parallel AB$ and $DC \parallel AB \Rightarrow AB \parallel EF \parallel DC$]

\therefore By the Converse of mid-point theorem, we can say that O is the mid-point of AC.

Now, in $\triangle CAB$, O is the mid-point of AC and $OF \parallel AB$.

\Rightarrow OF bisects BC [By Converse of mid-point theorem]

Or F is the mid-point of BC.

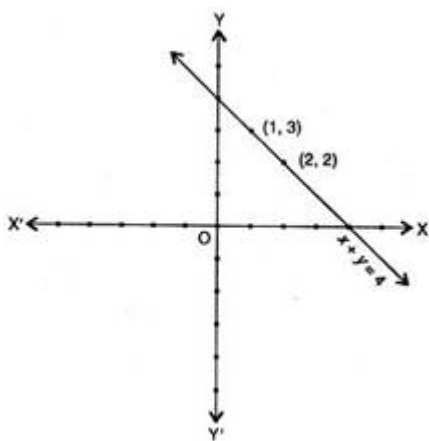
Hence proved

29. $x + y = 4$

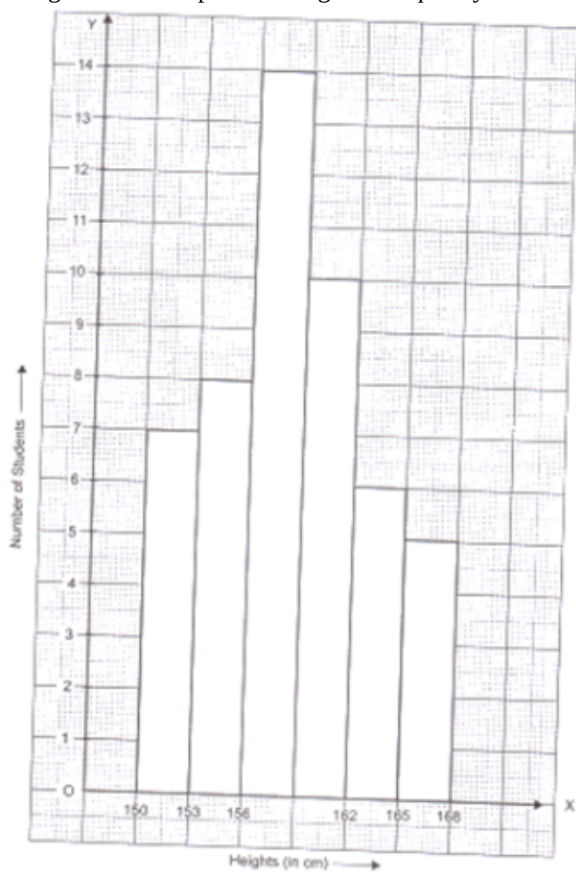
$\Rightarrow y = 4 - x$

x	1	2
y	3	2

We plot the points (1, 3) and (2, 2) on the graph paper and join the same by a ruler to get the line which is the graph of the equation $x + y = 4$.

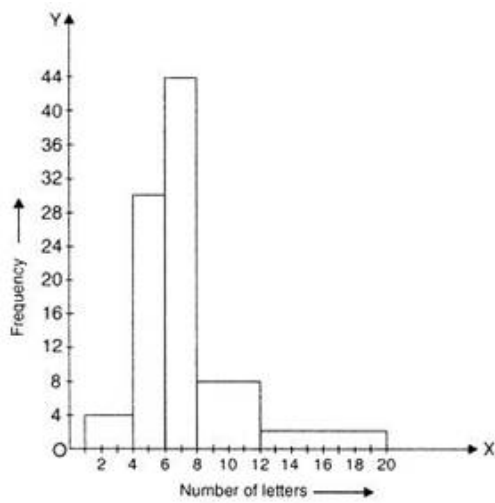


30. Histogram which represent the given frequency distribution is shown below:



OR

i.	Number of letters	Number of surnames	Width of the class	Length of the rectangle
	1-4	6	3	$\frac{6}{3} \times 2 = 4$
	4-6	30	2	$\frac{30}{2} \times 2 = 30$
	6-8	44	2	$\frac{44}{2} \times 2 = 44$
	8-12	16	4	$\frac{16}{4} \times 2 = 8$
	12-20	4	8	$\frac{4}{8} \times 2 = 1$



ii. The class interval in which the maximum number of surnames lie is 6-8.

31. The given polynomials are,

$$f(x) = ax^3 + 3x^2 - 3$$

$$p(x) = 2x^3 - 5x + a$$

Let,

R_1 is the remainder when $f(x)$ is divided by $x - 4$

$$\Rightarrow R_1 = f(4)$$

$$\Rightarrow R_1 = a(4)^3 + 3(4)^2 - 3$$

$$= 64a + 48 - 3$$

$$= 64a + 45 \dots(1)$$

Now, let

R_2 is the remainder when $p(x)$ is divided by $x - 4$

$$\Rightarrow R_2 = p(4)$$

$$\Rightarrow R_2 = 2(4)^3 - 5(4) + a$$

$$= 128 - 20 + a$$

$$= 108 + a \dots(2)$$

Given, $R_1 = R_2$

$$\Rightarrow 64a + 45 = 108 + a$$

$$\Rightarrow 63a = 63$$

$$\Rightarrow a = 1$$

This is the required value of a .

Section D

32. It is given that, $AB \parallel CD$ and t is a transversal

$$\therefore \angle AEF = \angle EFD \dots\dots(i) \text{ (Pair of alternate interior angles)}$$

EP is the bisectors of $\angle AEF$, (Given)

$$\therefore \angle AEP = \angle FEP = \frac{1}{2} \angle AEF$$

$$\Rightarrow \angle AEF = 2\angle FEP \dots\dots(ii)$$

Also, FQ is the bisectors of $\angle EFD$

$$\therefore \angle EFQ = \angle QFD = \frac{1}{2} \angle EFD$$

$$\Rightarrow \angle EFD = 2\angle EFQ \dots\dots(iii)$$

From equations (i), (ii) and (iii)

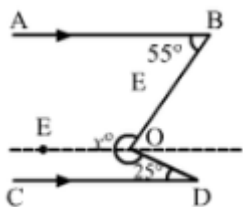
$$2\angle FEP = 2\angle EFQ$$

$$\Rightarrow \angle FEP = \angle EFQ$$

Thus, the lines EP and FQ are intersected by a transversal EF such that the pair of alternate interior angles formed are equal.

$\therefore EP \parallel FQ$ (If a transversal intersects two lines such that a pair of alternate interior angles are equal, then the two lines are parallel)

OR



Draw $EO \parallel AB \parallel CD$

Then, $\angle EOB + \angle EOD = x^\circ$

Now, $EO \parallel AB$ and BO is the transversal.

$\therefore \angle EOB + \angle ABO = 180^\circ$ [Consecutive Interior Angles]

$$\Rightarrow \angle EOB + 55^\circ = 180^\circ$$

$$\Rightarrow \angle EOB = 125^\circ$$

Again, $EO \parallel CD$ and DO is the transversal.

$\therefore \angle EOD + \angle CDO = 180^\circ$ [Consecutive Interior Angles]

$$\Rightarrow \angle EOD + 25^\circ = 180^\circ$$

$$\Rightarrow \angle EOD = 155^\circ$$

Therefore,

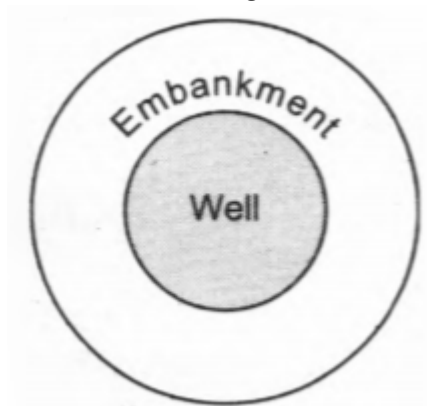
$$x^\circ = \angle EOB + \angle EOD$$

$$x^\circ = (125 + 155)^\circ$$

$$x^\circ = 280^\circ$$

33. Radius of the well, $r = 7\text{m}$, and its depth, $h = 15\text{ m}$.

Volume of the earth dugout



= volume of the well..

$$= (\pi r^2 h) \text{ cubic units.}$$

$$= \left(\frac{22}{7} \times 7 \times 7 \times 15 \right) \text{ m}^3 = 2310 \text{ m}^3$$

Now Width of the embankment = 7 m.

External radius of the embankment = $(7 + 7) \text{ m} = 14 \text{ m}$.

Internal radius of the embankment = 7 m.

Now Area of the embankment = $\pi \times [(14)^2 - 7^2] \text{ m}^2$

$$= \left[\frac{22}{7} \times (14 + 7) \times (14 - 7) \right] \text{ m}^2$$

$$= \left(\frac{22}{7} \times 21 \times 7 \right) \text{ m}^2 = 462 \text{ m}^2$$

Also Volume of the embankment = volume of the earth dug out = 2310 m^3

And Height of the embankment

$$= \left(\frac{\text{volume of the embankment in m}^3}{\text{area of the embankment in m}^2} \right) = \left(\frac{2310}{462} \right) \text{ m} = 5 \text{ m}$$

Hence, the height of the embankment formed = 5 m

34. Area (I) = area of isosceles triangle with $a = 1 \text{ cm}$ and $b = 5 \text{ cm}$

$$= \frac{a}{4} \sqrt{4b^2 - a^2}$$

$$= \frac{1}{4} \sqrt{100 - 1} = \frac{\sqrt{99}}{4} \text{ sq cm (approx)}$$

$$= 2.49 \text{ sq cm (approx.)}$$

Area (II) = area of rectangle with

$L = 6.5 \text{ cm}$ and $b = 1 \text{ cm}$

$$= 6.5 \times 1 \text{ sq cm} = 6.5 \text{ cm sq}$$

Area (III) = Area of trapezium

$$= 3 \times \text{Area of equilateral}$$

$$= 3 \times \frac{\sqrt{3}}{4} \times (1)^2 \text{ sq cm}$$

$$= \frac{3 \times 1.732}{4} \text{ or } \frac{5.196}{4} \text{ sq cm}$$

$$= 1.3 \text{ sq cm (approx.)}$$

$$\text{Area of (IV + V)} = 2 \times \frac{1}{2} \times 6 \times 1.5 \text{ sq cm} = 9 \text{ sq cm}$$

$$\therefore \text{total area of the paper used} = \text{Area (I+II+III+IV+V)}$$

$$= (2.49 + 6.5 + 1.3 + 9) \text{ sq cm}$$

$$= 19.29 \text{ sq cm}$$

OR

Let x and y be the two lines of the right \angle

$$\therefore AB = x \text{ cm, } BC = y \text{ cm and } AC = 10 \text{ cm}$$

\therefore By the given condition,

$$\text{Perimeter} = 24 \text{ cm}$$

$$x + y + 10 = 24 \text{ cm}$$

$$\text{Or } x + y = 14 \dots \text{(I)}$$

By Pythagoras theorem,

$$x^2 + y^2 = (10)^2 = 100 \dots \text{(II)}$$

$$\text{From (1), } (x + y)^2 = (14)^2$$

$$\text{Or } x^2 + y^2 + 2xy = 196$$

$$\therefore 100 + 2xy = 196 \text{ [From (II)]}$$

$$xy = \frac{96}{2} = 48 \text{ sq cm} \dots \text{(III)}$$

$$\text{Area of } \triangle ABC = \frac{1}{2}xy \text{ sq cm}$$

$$= \frac{1}{2} \times 48 \text{ sq cm}$$

$$= 24 \text{ sq cm} \dots \text{(IV)}$$

Again, we know that

$$(x - y)^2 = (x + y)^2 - 4xy$$

$$= (14)^2 - 4 \times 48 \text{ [From (I) \& (III)]}$$

$$\text{Or } x - y = \pm 2$$

$$\text{(i) When, } x - y = 2 \text{ and } x + y = 14, \text{ then } 2x = 16$$

$$\text{or } x = 8, y = 6$$

$$\text{(ii) When, } x - y = -2 \text{ and } x + y = 14, \text{ then } 2x = 12$$

$$\text{Or } x = 6, y = 8$$

Verification by using Heron's formula:

Sides are 6 cm, 8 cm and 10 cm

$$S = \frac{24}{2} = 12 \text{ cm}$$

$$\text{Area of } \triangle ABC = \sqrt{12(12 - 6)(12 - 8)(12 - 10)} \text{ sq cm}$$

$$= \sqrt{12 \times 6 \times 4 \times 2} \text{ sq cm}$$

$$= 24 \text{ sq cm}$$

Which is same as found in (IV)

Thus, the result is verified.

35. Let polynomial be

$$p(x) = x^3 - mx^2 - 13x + n$$

If $x - 1$ is factor of $p(x)$

$$\therefore p(1) = 0$$

$$(1)^3 - m(1)^2 - 13 \times 1 + n = 0$$

$$1 - m - 13 + n = 0$$

$$-m + n - 12 = 0$$

$$-12 = m - n \dots \dots \dots (1)$$

And if $x + 3$ is factor of $p(x)$

$$\therefore p(-3) = 0$$

$$(-3)^3 - m(-3)^2 - 13 \times (-3) + n = 0$$

$$-27 - 9m + 39 + n = 0$$

$$-9m + n + 12 = 0$$

$$12 - 9m + n = 0$$

$$12 = 9m - n \dots\dots(2)$$

Subtracting (1) from (2),

$$8m = 24$$

$$m = \frac{24}{8}$$

$$m = 3$$

Put $m = 3$ in (1),

$$3 - n = -12$$

$$-n = -12 - 3$$

$$-n = -15$$

$$n = 15$$

$$\therefore m = 3 \text{ and } n = 15$$

Section E

36. i. Let the no of questions whose answer is known to Ajay be x and number questions attempted by guessing be y .

$$x + y = 110$$

$$x + 14y = 80 \Rightarrow 4x + y = 320 \quad x + y = 110 \dots(1)$$

$$4x + y = 320 \dots(2)$$

Solving (1) and (2)

$$x + y - 4x - y = 110 - 320 = -210$$

$$\Rightarrow -3x = -210$$

$$\Rightarrow x = 70$$

ii. $x + y = 110$

$$x + 14y = 80 \Rightarrow 4x + y = 320$$

$$x + y = 110 \dots(1)$$

$$4x + y = 320 \dots(2)$$

Solving (1) and (2)

$$x + y - 4x - y = 110 - 320 = -210$$

$$\Rightarrow -3x = -210$$

$$\Rightarrow x = 70$$

Put $x = 70$ in (1)

$$70 + y = 110$$

$$\Rightarrow y = 40$$

40 question he answered by guessing.

iii. $70 - 40 \times \frac{1}{4} = 70 - 10 = 60$ marks

He scored 60 marks. $x - \frac{1}{4}(110 - x) = 95$

OR

$$\Rightarrow 4x - 110 + x = 380$$

$$\Rightarrow 5x = 380 + 110 = 490$$

$$\Rightarrow x = \frac{490}{5} = 98$$

So he answered 98 correct answers 12 by guessing.

37. i. In $\triangle APC$ and $\triangle BPC$

$$AP = BP \text{ (Given)}$$

$$CP = CP \text{ (common side)}$$

$$\angle ACP = \angle BCP = 90^\circ$$

By RHS criteria $\triangle APC \cong \triangle BPC$

ii. In $\triangle ACP$

$$\angle APC + \angle PAC + \angle ACP = 180^\circ$$

$$\Rightarrow x + 60^\circ + 90^\circ = 180^\circ \text{ (angle sum property of } \triangle)$$



$$\Rightarrow \angle x = 180^\circ - 150^\circ = 30^\circ$$

$$\angle x = 30^\circ$$

iii. In $\triangle APC$ and $\triangle BPC$

Corresponding part of congruent triangle

$$\angle X = \angle Y$$

$$\Rightarrow \angle Y = 30^\circ \text{ (given } \angle X = 30^\circ \text{)}$$

OR

In $\triangle APC$ and $\triangle BPC$

Corresponding part of congruent triangle

$$\angle PAC = \angle PBC$$

$$\Rightarrow \angle PBC = 60^\circ \text{ (given } \angle PAC = 60^\circ \text{)}$$

38. i. In $\triangle AOP$ and $\triangle BOP$

$$\angle APO = \angle BPO \text{ (Given)}$$

$$OP = OP \text{ (Common)}$$

$$AO = OB \text{ (radius of circle)}$$

$$\triangle AOP \cong \triangle BOP$$

$$AP = BP \text{ (CPCT)}$$

ii. In right $\triangle COQ$

$$CO^2 = OQ^2 + CQ^2$$

$$\Rightarrow 10^2 = 8^2 + CQ^2$$

$$\Rightarrow CQ^2 = 100 - 64 = 36$$

$$\Rightarrow CQ = 6$$

$$CD = 2CQ$$

$$\Rightarrow CD = 12 \text{ cm}$$

iii. In right $\triangle AOB$

$$AO^2 = OP^2 + AP^2$$

$$\Rightarrow 10^2 = 6^2 + AP^2$$

$$\Rightarrow AP^2 = 100 - 36 = 64$$

$$\Rightarrow AP = 8$$

$$AB = 2AP$$

$$\Rightarrow AB = 16 \text{ cm}$$

OR

There is one and only one circle passing through three given non-collinear points.